Resonant Frequencies of Implantable Piezoelectric Generators

Jessica G. Snyder[1]
University of Kansas, Lawrence, KS, 66044

and

Beth E. Lewandowski.[2]
NASA Glenn Research Center, Cleveland, OH,

[Abstract] Piezoelectric generators are currently being studied in a variety of energy harvesting situations. The goal of this project is to power implanted electronic medical devices with implanted piezoelectric generators, augmenting or replacing the external power supplies or implanted batteries currently used as power sources. Preliminary animal experiments have shown that this may be a viable method of powering internal electrical devices[2]. This paper covers the procedures used to determine the resonance frequencies of the piezoelectric generators in an effort to maximize the power output of the system.

Nomenclature

C_p	=	capacitance of piezoelectric stack
d	=	width of piezoelectric stack
f_{flex}	=	flexural resonance frequency
f_{long}	=	longitudinal resonance frequency
L	=	length of piezoelectric stack
n,m	=	positive integers
PZT	=	lead zirconate titanate
R_L	=	load resistor
v	=	speed of sound
V_p	=	voltage of piezoelectric stack

I. Introduction

PIEZOELECTRICS are materials which will produce a voltage when mechanically stressed and vice-a-versa. Types of piezoelectric materials fall into four main categories: naturally occurring crystals, man made crystals, man made ceramics and polymers. For these experiments, lead zirconate titanate stacks were used. Piezoelectric stacks are thin layers of capacitive materials connected electrically in parallel and mechanically in series. This configuration is most beneficial for this application since it produces the greatest capacitance per length of the stack and a compressive force applied to an end will transfer to all layers of the stack.

II. Problem Definition

The goal of this project was to determine the resonance frequencies of a piezoelectric stack generator in order to find the frequencies where the power output from the generator will be the greatest. Power output increases as impedance decreases but impedance across the stack varies non-linearly with frequency. The frequency design range for the implantable stacks is 0-50 Hz; however, frequencies up to 7 kHz were examined.

[1] NASA Academy Research Associate, jgsnyder21@hotmail.com
[2] Principle Investigator, Department of Biomedical Engineering, Mailstop 3-110, Beth.E.Lewandowski@nasa.gov

III. Experimental Set-up

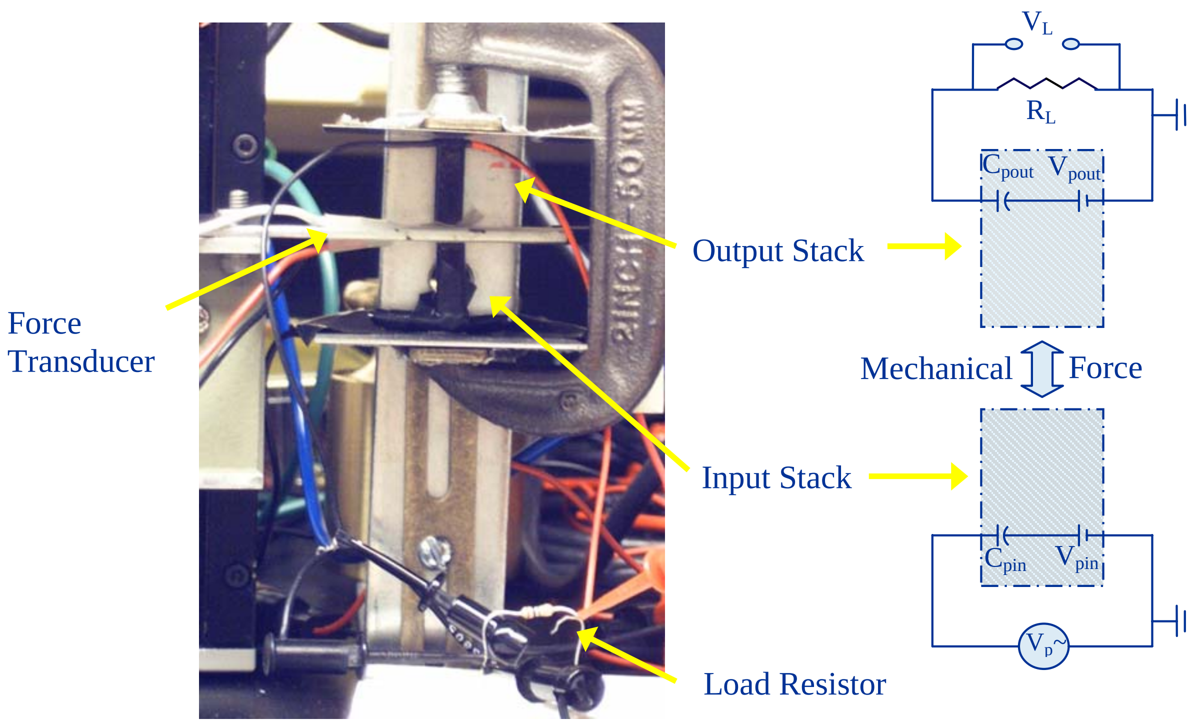

Figure 1: **Photo (left) and schematic (right) of experimental set-up**

Two piezoelectric stacks were used for this procedure. One stack served as the actuator and was driven with a sin wave from a frequency generator. The second stack was the generator and was clamped together with the actuator so that force from the actuator would be transmitted to the generator, see Figure 1. Since the impedance of the stack can not be measured directly, voltage across a load resistor was measured over the given range of frequencies. Piezoelectric stacks are represented electrically as a capacitance (C_p) and voltage (V_p) connected in series. The capacitance, and consequently the impedance, varies with frequency. V_p is proportional to the force applied to the stack.

Initially, it was attempted to place a force transducer between the two stacks in order to determine the magnitude of the transferred force which would enable the calculation of the impedance. However, no available force transducer was precise enough to detect the force. Due to this, it was only possible to analyze the voltage response of the stacks as the frequency varied.

Results

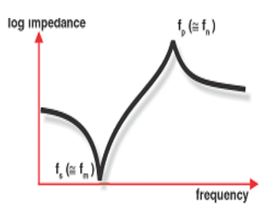

Figure 2. Expected theoretical response of stack impedance[1]

The trend of impedance in a piezoelectric as it approaches resonance is fairly well characterized as function of frequency. Figure 2 shows this trend. Resonance occurs when impedance is the lowest. Just after reaching resonance, the impedance will reach a maximum at the anti-resonance of the piezoelectric.

The first four runs of the experiment are shown averaged in Figure 3. There are a few dips and peaks which might be resonances, however the overall trend of the voltage amplitude was not expected to be decreasing, but in fact the opposite.

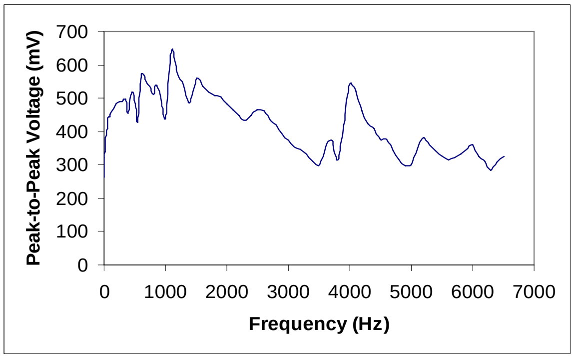

Figure 3. Initial results of output voltage amplitude vs. frequency

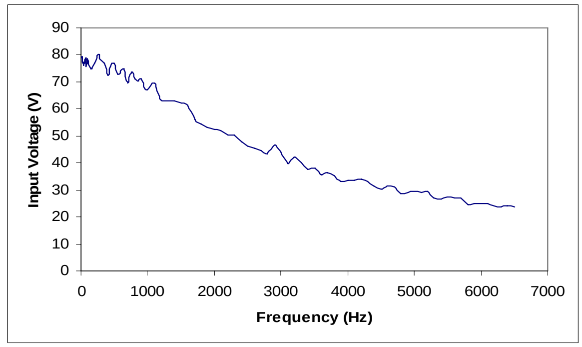

Figure 4. Input voltage amplitude as it varies with frequency.

After noticing this, it was determined that the input voltage amplitude, Figure 4, was decreasing exponentially as frequency increased. The most likely cause for this behavior was due to the setup of frequency generator and actuating piezoelectric which together formed a low pass filter, effectively decreasing the output voltage as a function of frequency. The remedy for this was to manually adjust the output voltage from the frequency generator in order to keep it constant. Figure 6**Error! Reference source not found.** shows the results when the input voltage was kept at a constant. The overall trend of the voltage increased as expected and the possible resonant peaks became more evident.

The first possible resonance is shown in Figure 5 and occurred at 370 Hz. Two more possible resonances occurred at 940 and 2150 Hz. The most dramatic resonance peak occurred at 3600 Hz.

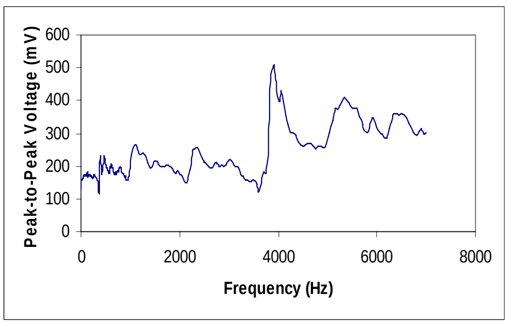

Figure 6. Output voltage amplitude vs frequency with constant input voltage amplitude

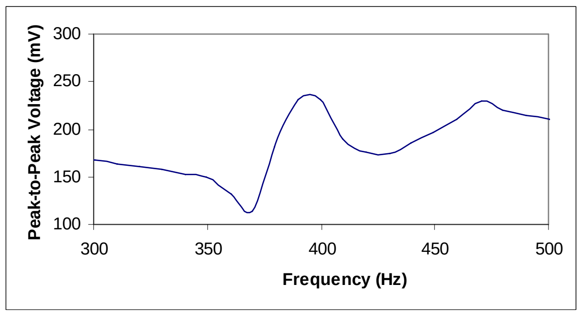

Figure 5. First possible instance of

Several methods were attempted to validate the experimental results. According to the piezoelectric stack manufacturers, the only resonant frequency for the stacks used is at 74 kHz. An impedance analyzer was then used to test the stacks but now resonance peaks were detected under 7 kHz. An attempt was also made to validate the results mathematically using equations for modal resonant frequencies. According to theory, piezoelectric stacks can produce resonances in two manners, longitudinally or flexurally[3]. Modes of longitudinal resonance should occur according to the relationship

$$f_n = n\frac{v}{2L} \qquad \textbf{Equation 1}$$

while flexural resonance should occur according to

$$f_n \approx (2n+1)^2 \frac{\pi v d}{16\sqrt{3}\cdot L^2} \qquad \textbf{Equation 2}$$

For both equations, n is a positive integer, v is the speed of sound in the material, L is the length of the rod, and d is the side length. By dividing the two equations, the unknown speed of sound is removed from the equation and the experimentally found resonance peaks can be used to determine the number of the mode for each type of resonance.

$$\frac{f_{n,long}}{f_{m,\,flex}} = \frac{8\sqrt{3}\cdot nL}{\pi d\,(2m+1)^2} \qquad \textbf{Equation 3}$$

Table 1: Physical stack characteristics and proposed resonant frequencies

Variable	Value	Units
L	0.018	m
d	0.005	m
f1	369	Hz
f2	970	Hz
f3	2151	Hz
f4	3603	Hz

Using $\frac{f_{n,long}}{f_{m,\,flex}} = \frac{8\sqrt{3}\cdot nL}{\pi d\,(2m+1)^2}$ Equation 3 and the values from Table 1, ratios of the possible modes were developed, shown in Table 2. The table was produced by assuming that each of the possible resonances was the first flexural mode, then that each of the last three possible resonances was the second flexural mode. The value for n was calculated using $\frac{f_{n,long}}{f_{m,\,flex}} = \frac{8\sqrt{3}\cdot nL}{\pi d\,(2m+1)^2}$ Equation 3. Any ratios which resulted in nearly whole values for n are noted.

Table 2: Ratios of possible resonant frequencies

	m=1		M=2	
f_long/f_flex	**n**		**n**	
f_1/f_2	0.216		0.599	
f_1/f_3	0.097		0.270	
f_1/f_4	0.058		0.161	
f_2/f_1	1.490		n/a	
f_2/f_3	0.256		0.710	
f_2/f_4	0.153		0.424	
f_3/f_1	3.304		n/a	
f_3/f_2	1.257	f_3=1st long; f_2=1st flex	3.491	
f_3/f_4	0.338		0.940	f_3=1st long; f_4=2nd flex
f_4/f_1	5.534		n/a	
f_4/f_2	2.105	f_4=2nd long; f_2=1st flex;	5.848	
f_4/f_3	0.949	f_4=1st long; f_3=1st flex	2.637	

Although several of the ratios result in modes occurring at the second, third and fourth frequencies, none included the first frequency. Moving on from this, the possibility was addressed that all four frequencies may be of the same form, either flexural or longitudinal. Using $f_n = n\frac{v}{2L}$ Equation 1 and $f_n \approx (2n+1)^2 \frac{\pi v d}{16\sqrt{3}\cdot L^2}$ Equation 2 and solving for the speed of sound, it was assumed that if all four frequencies fit into one form of resonance, a similar speed of sound would be calculated for all frequencies. The results of this method are shown in

Table 3: Results assuming four frequencies are either longitudinal or flexural

	Speed of Sound (*v*) m/s	
Frequency (Hz)	**Longitudinal**	**Flexural**
370	13.32	23.50
950	16.92	25.09
2151	25.81	25.09
3600	32.40	25.41
Average	22.11	23.93
Percent error	39.06%	7.07%

Judging by percent error in the two groups, it seems possible that the four frequencies may be flexural resonances. However, this would produce a speed of sound through the stack of only 23.93 m/s.

IV. Conclusion

A possible explanation for the difference in results between the mechanical stimulation of the two stack set-up and the electrical stimulation of the impedance analyzer is that the former was performed under high field conditions while the was executed under low field conditions. This suggests that there may be a relationship between voltage and resonance at any given frequency. The results of the mathematical approach did not yield a reasonable value for the speed of sound, but the fact that the values are close together suggests a relationship and should be investigated further.

Acknowledgments

J.G. Snyder thanks B.E. Lewandowski for her guidance and encouragement during this project, the NASA Glenn Academy for providing the opportunity, and the Kansas Space Grant Consortium for funding.

References

[1]APC International, Ltd. *Determining Resonance Frequency.* www.americanpiezo.com/piezo_theory/resonance_frequency.html

[2]Lewandowski, B.E., K.L. Kilgore, and K.J. Gustafson. *Design Considerations for an Implantable, Muscle Powered Piezoelectric System for Generating Electrical Power*. Annals of Biomedical Engineering, 2007. 35(1): p.631-641

[3]Tremblay, D.L., "Resonance in a piezoelectric material," Physics Department, The College of Wooster, Wooster, OH, 2006